A First Course on Kinetics and Reaction Engineering

Class 7 on Unit 7


## Where We've Been

- Part I - Chemical Reactions
- Part II - Chemical Reaction Kinetics
- A. Rate Expressions
- 4. Reaction Rates and Temperature Effects
- 5. Empirical and Theoretical Rate Expressions
- 6. Reaction Mechanisms
- 7. The Steady State Approximation
- 8. Rate Determining Step
- 9. Homogeneous and Enzymatic Catalysis
- 10. Heterogeneous Catalysis
- B. Kinetics Experiments
- C. Analysis of Kinetics Data
- Part III-Chemical Reaction Engineering
- Part IV - Non-Ideal Reactions and Reactors


## Mechanistic Rate Expressions

$$
r_{i, j}=\sum_{\substack{s=\text { all } \\ \text { steps }}} v_{i, s}\left(k_{s, f} \prod_{\substack{m=a l l \\ \text { reactants }}}[m]^{-v_{m, s}}-k_{s, r} \prod_{\substack{n=\text { all } \\ \text { products }}}[n]^{v_{n, s}}\right)
$$

- If a mechanistic step is kinetically insignificant, delete the terms that correspond to its rate
- If a mechanistic step is effectively irreversible, delete the term that corresponds to its reverse rate
- Apply Bodenstein steady state approximation to each reactive intermediate

$$
0=\sum_{\substack{s=\text { all } \\ \text { steps }}} V_{i, s}\left(k_{s, f} \prod_{\substack{m=\text { all } \\ \text { reactants }}}[m]^{-v_{m, s}}-k_{s, r} \prod_{\substack{n=\text { all } \\ \text { products }}}[n]^{v_{n, s}}\right)
$$

- Solve the resulting set of equations to obtain expressions for the concentrations of the reactive intermediates in terms of concentrations of stable species and rate coefficients from the reaction mechanism
- Substitute these expressions into the mechanistic rate expression


## Questions?

## The Bodenstein Steady State Approximation

From a macroscopic point of view, the decomposition of $\mathrm{N}_{2} \mathrm{O}_{5}$ appears to proceed according to equation (1). In actuality, that reaction is nonelementary. Suppose that the mechanism is given by reactions (2) through (4), where reaction (2) is reversible, but reactions (3) and (4) are irreversible. Using the Bodenstein steady state approximation, derive a rate expression for reaction (1) with respect to $\mathrm{O}_{2}$. The rate expression should not contain concentrations of reactive intermediates.

$$
\begin{align*}
& 2 \mathrm{~N}_{2} \mathrm{O}_{5} \rightleftarrows 4 \mathrm{NO}_{2}+\mathrm{O}_{2}  \tag{1}\\
& \mathrm{~N}_{2} \mathrm{O}_{5} \rightleftarrows \mathrm{NO}_{2}+\mathrm{NO}_{3}  \tag{2}\\
& \mathrm{NO}_{2}+\mathrm{NO}_{3} \rightarrow \mathrm{NO}_{2}+\mathrm{NO}+\mathrm{O}_{2}  \tag{3}\\
& \mathrm{NO}+\mathrm{NO}_{3} \rightarrow 2 \mathrm{NO}_{2} \tag{4}
\end{align*}
$$

## Approach

- Check that the mechanism is valid
- Identify the stable species and the reactive intermediates
- Write an expression for the overall rate with respect to one of the reactants or products of the apparent overall reaction
- Simplify the rate expression if any of the steps are kinetically insignificant or effectively irreversible
- Write the Bodenstein steady state approximation for each reactive intermediate
- Simplify the equations if any of the steps are kinetically insignificant or effectively irreversible
- Solve the resulting equations to get expressions for the concentrations (or partial pressures) of each of the reactive intermediates
- The resulting expressions should only contain rate coefficients and concentrations of stable species
- Anywhere that the concentration of a reactive intermediate appears in the rate expression for the overall reaction, substitute the expression for it that resulted from applying the Bodenstein steady state approximation
- Simplify, if possible


## Check that the Mechanism is Valid

- Macroscopically observed reaction

$$
\begin{equation*}
2 \mathrm{~N}_{2} \mathrm{O}_{5} \rightleftarrows 4 \mathrm{NO}_{2}+\mathrm{O}_{2} \tag{1}
\end{equation*}
$$

- Proposed mechanism

$$
\begin{equation*}
\mathrm{N}_{2} \mathrm{O}_{5} \rightleftarrows \mathrm{NO}_{2}+\mathrm{NO}_{3} \tag{2}
\end{equation*}
$$

$$
\begin{align*}
& \mathrm{NO}_{2}+\mathrm{NO}_{3} \rightarrow \mathrm{NO}_{2}+\mathrm{NO}+\mathrm{O}_{2}  \tag{3}\\
& \mathrm{NO}+\mathrm{NO}_{3} \rightarrow 2 \mathrm{NO}_{2} \tag{4}
\end{align*}
$$

## Identify the Stable Species and the Reactive Intermediates

- Macroscopically observed reaction

$$
\begin{equation*}
2 \mathrm{~N}_{2} \mathrm{O}_{5} \rightleftarrows 4 \mathrm{NO}_{2}+\mathrm{O}_{2} \tag{1}
\end{equation*}
$$

- Proposed mechanism
$\mathrm{N}_{2} \mathrm{O}_{5} \rightleftarrows \mathrm{NO}_{2}+\mathrm{NO}_{3}$

$$
\begin{align*}
& \mathrm{NO}_{2}+\mathrm{NO}_{3} \rightarrow \mathrm{NO}_{2}+\mathrm{NO}+\mathrm{O}_{2}  \tag{3}\\
& \mathrm{NO}+\mathrm{NO}_{3} \rightarrow 2 \mathrm{NO}_{2}
\end{align*}
$$

- The mechanism is valid
- Reaction (1) $=2 \mathrm{x}$ reaction (2) + reaction (3) + reaction (4)


## Write an Expression for the Overall Rate and Simplify it, if Possible

- Macroscopically observed reaction

$$
\begin{equation*}
2 \mathrm{~N}_{2} \mathrm{O}_{5} \rightleftarrows 4 \mathrm{NO}_{2}+\mathrm{O}_{2} \tag{1}
\end{equation*}
$$

- Proposed mechanism

$$
\begin{equation*}
\mathrm{N}_{2} \mathrm{O}_{5} \rightleftarrows \mathrm{NO}_{2}+\mathrm{NO}_{3} \tag{2}
\end{equation*}
$$

$$
\begin{align*}
& \mathrm{NO}_{2}+\mathrm{NO}_{3} \rightarrow \mathrm{NO}_{2}+\mathrm{NO}+\mathrm{O}_{2}  \tag{3}\\
& \mathrm{NO}+\mathrm{NO}_{3} \rightarrow 2 \mathrm{NO}_{2} \tag{4}
\end{align*}
$$

- The mechanism is valid
- Reaction (1) $=2 x$ reaction (2) + reaction (3) + reaction (4)
- The reactive intermediates are NO and $\mathrm{NO}_{3}$


## Apply the Bodenstein Steady State Approximation to the Reactive Intermediates and Simplify

- Macroscopically observed reaction

$$
\begin{equation*}
2 \mathrm{~N}_{2} \mathrm{O}_{5} \rightleftarrows 4 \mathrm{NO}_{2}+\mathrm{O}_{2} \tag{1}
\end{equation*}
$$

- Proposed mechanism

$$
\begin{align*}
& \mathrm{N}_{2} \mathrm{O}_{5} \rightleftarrows \mathrm{NO}_{2}+\mathrm{NO}_{3}  \tag{2}\\
& \mathrm{NO}_{2}+\mathrm{NO}_{3} \rightarrow \mathrm{NO}_{2}+\mathrm{NO}+\mathrm{O}_{2}  \tag{3}\\
& \mathrm{NO}+\mathrm{NO}_{3} \rightarrow 2 \mathrm{NO}_{2} \tag{4}
\end{align*}
$$

- The mechanism is valid
- Reaction (1) $=2 \times$ reaction (2) + reaction (3) + reaction (4)
- The reactive intermediates are NO and $\mathrm{NO}_{3}$
- Overall rate with respect to $\mathrm{O}_{2}$

$$
r_{O_{2}, 1}=\left(k_{3, f}\left[\mathrm{NO}_{2}\right]\left[\mathrm{NO}_{3}\right]-k_{3, r}\left[\mathrm{NO}_{2}\right][\mathrm{NO}]\left[\mathrm{O}_{2}\right]\right)
$$

- Reactions (3) and (4) are irreversible, so the terms associated with their reverse rate can be deleted

$$
\text { , } r_{O_{2}, 1}=k_{3, f}\left[N O_{2}\right]\left[N O_{3}\right]
$$

- This rate expression is not acceptable because the concentration of $\mathrm{NO}_{3}$, a reactive intermediate, appears in it


## Solve for Expressions for the Reactive Intermediates

- Bodenstein steady state approximation for NO and $\mathrm{NO}_{3}$

$$
\begin{aligned}
-0= & \left(k_{3, f}\left[N O_{2}\right]\left[\mathrm{NO}_{3}\right]-k_{3, r}\left[N O_{2}\right][N O]\left[O_{2}\right]\right)-\left(k_{4, f}[N O]\left[\mathrm{NO}_{3}\right]-k_{4, r}\left[\mathrm{NO}_{2}\right]^{2}\right) \\
\cdot 0 & =\left(k_{2, f}\left[\mathrm{~N}_{2} O_{5}\right]-k_{2, r}\left[N O_{2}\right]\left[N O_{3}\right]\right)-\left(k_{3, f}\left[N O_{2}\right]\left[N O_{3}\right]-k_{3, r}\left[N O_{2}\right][N O]\left[O_{2}\right]\right) \\
& -\left(k_{4, f}[N O]\left[N O_{3}\right]-k_{4, r}\left[N O_{2}\right]^{2}\right)
\end{aligned}
$$

- Reactions (3) and (4) are irreversible, so the terms associated with their reverse rate can be deleted
- $0=k_{3, f}\left[\mathrm{NO}_{2}\right]\left[\mathrm{NO}_{3}\right]-k_{4, f}[\mathrm{NO}]\left[\mathrm{NO}_{3}\right]$
- $0=k_{2, f}\left[N_{2} \mathrm{O}_{5}\right]-k_{2, r}\left[\mathrm{NO}_{2}\right]\left[\mathrm{NO}_{3}\right]-k_{3, f}\left[\mathrm{NO}_{2}\right]\left[\mathrm{NO}_{3}\right]-k_{4, f}[\mathrm{NO}]\left[\mathrm{NO}_{3}\right]$


## Substitute into the Overall Rate Expression

- Bodenstein steady state approximation for NO and $\mathrm{NO}_{3}$

$$
\begin{aligned}
-0= & \left(k_{3, f}\left[\mathrm{NO}_{2}\right]\left[\mathrm{NO}_{3}\right]-k_{3, r}\left[\mathrm{NO}_{2}\right][\mathrm{NO}]\left[\mathrm{O}_{2}\right]\right)-\left(k_{4, f}[\mathrm{NO}]\left[\mathrm{NO}_{3}\right]-k_{4, r}\left[\mathrm{NO}_{2}\right]^{2}\right) \\
-0= & \left(k_{2, f}\left[\mathrm{~N}_{2} \mathrm{O}_{5}\right]-k_{2, r}\left[\mathrm{NO}_{2}\right]\left[\mathrm{NO}_{3}\right]\right)-\left(k_{3, f}\left[\mathrm{NO}_{2}\right]\left[\mathrm{NO}_{3}\right]-k_{3, r}\left[N O_{2}\right][\mathrm{NO}]\left[O_{2}\right]\right) \\
& -\left(k_{4, f}[N O]\left[N O_{3}\right]-k_{4, r}\left[N O_{2}\right]^{2}\right)
\end{aligned}
$$

- Reactions (3) and (4) are irreversible, so the terms associated with their reverse rate can be deleted
- $0=k_{3, f}\left[\mathrm{NO}_{2}\right]\left[\mathrm{NO}_{3}\right]-k_{4, f}[\mathrm{NO}]\left[\mathrm{NO}_{3}\right]$
- $0=k_{2, f}\left[N_{2} \mathrm{O}_{5}\right]-k_{2, r}\left[\mathrm{NO}_{2}\right]\left[\mathrm{NO}_{3}\right]-k_{3, f}\left[\mathrm{NO}_{2}\right]\left[\mathrm{NO}_{3}\right]-k_{4, f}[\mathrm{NO}]\left[\mathrm{NO}_{3}\right]$
- Solving for the concentrations of NO and $\mathrm{NO}_{3}$
- $[\mathrm{NO}]=\frac{k_{3, f}}{k_{4, f}}\left[\mathrm{NO}_{2}\right]$
$-\left[\mathrm{NO}_{3}\right]=\frac{k_{2, f}\left[\mathrm{~N}_{2} \mathrm{O}_{5}\right]}{\left(k_{2, r}+2 k_{3, f}\right)\left[N O_{2}\right]}$


## Solution

- Bodenstein steady state approximation for NO and $\mathrm{NO}_{3}$

$$
\begin{aligned}
-0= & \left(k_{3, f}\left[\mathrm{NO}_{2}\right]\left[\mathrm{NO}_{3}\right]-k_{3, r}\left[\mathrm{NO}_{2}\right][\mathrm{NO}]\left[\mathrm{O}_{2}\right]\right)-\left(k_{4, f}[\mathrm{NO}]\left[\mathrm{NO}_{3}\right]-k_{4, r}\left[\mathrm{NO}_{2}\right]^{2}\right) \\
-0= & \left(k_{2, f}\left[\mathrm{~N}_{2} \mathrm{O}_{5}\right]-k_{2, r}\left[\mathrm{NO}_{2}\right]\left[\mathrm{NO}_{3}\right]\right)-\left(k_{3, f}\left[\mathrm{NO}_{2}\right]\left[\mathrm{NO}_{3}\right]-k_{3, r}\left[N O_{2}\right][\mathrm{NO}]\left[\mathrm{O}_{2}\right]\right) \\
& -\left(k_{4, f}[N O]\left[\mathrm{NO}_{3}\right]-k_{4, r}\left[\mathrm{NO}_{2}\right]^{2}\right)
\end{aligned}
$$

- Reactions (3) and (4) are irreversible, so the terms associated with their reverse rate can be deleted
- $0=k_{3, f}\left[\mathrm{NO}_{2}\right]\left[\mathrm{NO}_{3}\right]-k_{4, f}[\mathrm{NO}]\left[\mathrm{NO}_{3}\right]$
- $0=k_{2, f}\left[N_{2} \mathrm{O}_{5}\right]-k_{2, r}\left[\mathrm{NO}_{2}\right]\left[\mathrm{NO}_{3}\right]-k_{3, f}\left[\mathrm{NO}_{2}\right]\left[\mathrm{NO}_{3}\right]-k_{4, f}[\mathrm{NO}]\left[\mathrm{NO}_{3}\right]$
- Solving for the concentrations of NO and $\mathrm{NO}_{3}$
- $[\mathrm{NO}]=\frac{k_{3, f}}{k_{4, f}}\left[\mathrm{NO}_{2}\right]$
- $\left[\mathrm{NO}_{3}\right]=\frac{k_{2, f}\left[\mathrm{~N}_{2} \mathrm{O}_{5}\right]}{\left(k_{2, r}+2 k_{3, f}\right)\left[\mathrm{NO}_{2}\right]}$
- Substituting into the rate expression

$$
r_{O_{2}, 1}=k_{3, f}\left[N O_{2}\right]\left[N O_{3}\right]=\frac{k_{2, f} k_{3, f}}{\left(k_{2, r}+2 k_{3, f}\right)}\left[N_{2} O_{5}\right]
$$

## Non-Elementary Rate Expressions with Respect to Different Species

$$
\begin{equation*}
\mathrm{H}_{2}+\mathrm{Br}_{2} \rightleftarrows 2 \mathrm{HBr} \tag{1}
\end{equation*}
$$

- Mechanism

$$
\begin{align*}
& \mathrm{Br}_{2} \rightleftarrows 2 \mathrm{Br} \cdot  \tag{2}\\
& \mathrm{Br} \cdot+\mathrm{H}_{2} \rightleftarrows \mathrm{HBr}+\mathrm{H} \cdot  \tag{3}\\
& \mathrm{H} \cdot+\mathrm{Br}_{2} \rightleftarrows \mathrm{HBr}+\mathrm{Br} \cdot  \tag{4}\\
& 2 \mathrm{H} \cdot \rightleftarrows \mathrm{H}_{2} \tag{5}
\end{align*}
$$

- In Example 7.1 the rate of reaction (1) with respect to $\mathrm{H}_{2}$ was used
- Simplified assuming step (4) to be effectively irreversible and step 5 to be kinetically insignificant
- Applied the Bodenstein steady state approximation to $\mathrm{Br} \cdot$ and $\mathrm{H} \cdot$

$$
P_{H \cdot}=\frac{k_{3, f} P_{H_{2}} \sqrt{\frac{k_{2, f} P_{B r_{2}}}{k_{2, r}}}}{k_{3, r} P_{H B r}+k_{4, f} P_{B r_{2}}} \quad P_{B r \cdot}=\sqrt{\frac{k_{2, f} P_{B r_{2}}}{k_{2, r}}}
$$

- Final rate expression after substitution: $r_{H_{2}, 1}=-\frac{k_{3, f} k_{4, f} \sqrt{\frac{k_{2, f}}{k_{2, r}}} P_{H_{2}} P_{B r_{2}}^{3 / 2}}{k_{3, r} P_{H B r}+k_{4, f} P_{B r_{2}}}$
- Half of the class will repeat only using the rate of reaction (1) with respect to $\mathrm{Br}_{2}$ and half using the rate of reaction (1) with respect to HBr


## Final Rate Expressions are Identical

- The Bodenstein steady state approximation does not change, so the expressions for the partial pressures of $\mathrm{Br} \cdot$ and $\mathrm{H} \cdot$ remain the same

$$
P_{H \cdot}=\frac{k_{3, f} P_{H_{2}} \sqrt{\frac{k_{2, f} P_{B r_{2}}}{k_{2, r}}}}{k_{3, r} P_{H B r}+k_{4, f} P_{B r_{2}}} \quad \quad P_{B r \cdot}=\sqrt{\frac{k_{2, f} P_{B r_{2}}}{k_{2, r}}}
$$

- Rates with respect to $\mathrm{Br}_{2}$ and HBr (after simplification for irreversible and insignificant steps)

$$
\begin{aligned}
& r_{B r_{2}, 1}=-\left(k_{2, f} P_{B r_{2}}-k_{2, r} P_{B r \cdot}^{2}\right)-\left(k_{4, f} P_{H \cdot} P_{B r_{2}}\right) \\
& r_{H B r, 1}=\left(k_{3, f} P_{B r \cdot} \cdot P_{H_{2}}-k_{3, r} P_{H B r} P_{H \cdot}\right)+\left(k_{4, f} P_{H \cdot} \cdot P_{B r_{2}}\right)
\end{aligned}
$$

- After substitution of the Bodenstein steady state approximation expressions for the concentrations of the reactive intermediates

$$
r_{1}=\frac{r_{H_{2}, 1}}{-1}=\frac{r_{B r_{2}, 1}}{-1}=\frac{r_{H B r, 1}}{2}=\frac{k_{3, f} k_{4, f} \sqrt{\frac{k_{2, f}}{k_{2, r}}} P_{H_{2}} P_{B r_{2}}^{3 / 2}}{k_{3, r} P_{H B r}+k_{4, f} P_{B r_{2}}}
$$

## Where We're Going

- Part I - Chemical Reactions
- Part II - Chemical Reaction Kinetics
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